PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

MASTER'S DEGREE PROGRAM OFFER ACADEMIC TRACK



University of Jijel

Field : Mathématiques

Major : Mathématiques

Specialization : E.D.P et applications

I – Master's Identity Sheet

1 - Training Location:

Faculty: Exact Sciences and Computer Science Department: Mathematics

2 - Training Partners:

- Other universities:
- Companies and other socio-economic partners:
- International partners:

3 - Context and Objectives of the Program

A – Admission Requirements

Bachelor's degree (LMD system) after application review. Bachelor's degree and DES from the classical system after application review by the teaching team.

B – Program Objectives

The main objective of this Master's program is to train students in mathematical engineering to enable their professional integration in the industrial sector. Indeed, it is relevant to offer such a program given the growing importance of mathematics in the modeling of physical phenomena in industry and everyday social sciences.

Moreover, many existing Master's programs in mathematics in Algerian universities are oriented toward fundamental research and teaching. Through this Master's program, we aim to bring a new and interesting specificity to mathematics in the industrial field.

The program aims to provide students with:

• A training focused on the application of mathematics for the numerical resolution of physical models.

• A theoretical foundation allowing students to engage in research activities within a doctoral program or join the R&D sector in industry.

• A training in mathematical engineering enabling professional integration in the industrial field.

C – Target Profiles and Professional Skills

This Master's program aims to train PhD candidates in applied mathematics, particularly in the fields of ODEs, PDEs, numerical analysis, and modeling.

The targeted applications are mainly the numerical resolution of equations arising from physics (structural analysis, flow problems, wave equations, magnetic resonance, etc.), as well as certain applications related to optimal control and dynamic systems.

The goal is to combine computer tools with mathematics to train professionals in mathematical engineering and R&D in the industrial sector.

D – Regional and National Employability Opportunities

Employment opportunities in:

- Engineering in the industrial sector
- Research and development within the industrial sector
- Higher education and scientific research via doctoral training

E – Bridges to Other Specialties

Students may change their study path at the beginning of the second semester (S2) with the approval of the teaching team.

F – **Program Monitoring Indicators**

The teaching team ensures program monitoring by regularly organizing pedagogical committees and producing a semi-annual evaluation report.

G – Supervision Capacity

Given the staff associated with the Master's program in PDEs and Applications, it is possible to supervise up to 30 students.

II Semester organisation sheet for teaching <u>First semester</u>

Teaching Unit	Hours (15 weeks)	a lectur e	exerci ses	works Applie d	Perso nal work	Factors	points	continuous	exam
basic units									
UEF1 (mandatory)							18		
Functional analysis	45,0 hours	1h30	1h30		1 hour	2	4	Х	Х
Distributions and Fourier analysis	45,0 hours	3 hours	1h30		1 hour	3	5	Х	Х
UEF2(compulsory(
Dynamic systems and optimal control	67,5 hours	1h30	1h30			2	4	х	X
Mathematical Modeling	45 hours	1h30	1h30			2	5	Х	X
Curriculum units									
UEM1(compulsory)							9		
Matrix Analysis	60,0 hours	1h30	1h30	1h30		2	6	Х	Х
Matlab (PDE tool box)	45,0 hours	1h30		1h30	1 hour	1	3		X
Discovery units									
UED1(compulsory)							3		
Scientific English	45 hours	1h30	1h30			2	3	Х	Х
the total	375,0	10,5	7,5	3,0	2,5		15		

Second semester

Teaching Unit	Hours (15 weeks)	a lecture	exercises	works Applied	Personal work	Factors	points	continuous	exam
Fondamental units	203 90	6 3	6 3	1,5 0	1,5 0		9 5		
UEF3(compulsory)	45 45	1,5 1,5	1,5 1,5				3 2		
Sobolev Spaces	203 90	6 3	6 3	1,5 0	1,5 0		9 5	Х	х
E.D.P of evolution	45	1,5	1,5				3	Х	х
UEF4(compulsory)	113	3	3	1,5	1,5		4		
Numerical analysis of ODE	60h	1h30	1h30		1 h	2	5	Х	х
Introduction to Unbounded Operators	45h	1h30	1h30			2	4	Х	х
Methodology units									
UEM2(compulsory)	105	3	1,5	1,5	1		3		
Introduction to Variational Approximation Methods	60	1,5	1,5	1,5	1		2 1	Х	X
IT Tools	45	1,5	1,5				2		Х
Discovery units									
UED2(compulsory)	67,5	3	1,5	0	0		3		
Analytical mechanics	45	3h	1h30	1 h		3	3	v	V
Scientific English	22,5	1,5				1	1	Х	Х
the total	375	12	9	3	2,5		15	30	

Third semester

Teaching Unit	Hours (15	-	Tutorial	works	Personal	Factors	points	continuous	exam
	weeks)	Lecture		Applied	work		L		
Fondamental units	202,5	6	6	1,5	0		10		
UEF5(compulsory)	90	3	3				5		
Variational analysis of PDEs	45 45	1,5 1,5	1,5 1,5				3 2	Х	X
Variational inequalities	45	1,5	1,5				3	Х	х
UEF6(compulsory)	112,5	3	3	1,5	0				
Finite element method	45h	1h30	1h30			2	6	Х	X
Spectral methods									
UEM3(compulsory)	105	3	1,5	1,5	1		9		
Methodology	75h	1h30	1h30			2	6	Х	Х
Numerical analysis of PDEs	52h 30	1h30		2 h		2	3		X
Discovery units									
Interjection Units									
UET3(compulsory)	67,5	3	0	0	0		1		
Scientific English	45H	1.5				1	1		
Labour legislation and ethics	22.5H	1.5				1			
the total	375	12	7,5	3	1	15	30		

Semester 4: Research work sanctioned by a dissertation and a defense with the possibility of Internship in an engineering company

	VHS	FACTOR	PIONTS
PERSONAL WORK	322	14	30
Stage			
THESIS			
Semester 4	322	14	30

Semester 1:

Functional analysis

Distributions and Fourier analysis

Dynamic systems and optimal control

Mathematical Modeling

Matrix Analysis

Matlab (PDE tool box)

Scientific English

Semester 2:

Sobolev Spaces

E.D.P of evolution

Numerical analysis of ODE

Introduction to Unbounded Operators

Introduction to Variational Approximation Methods

IT Tools

Analytical mechanics

Scientific English

Semester 3:

Variational analysis of PDEs

Variational inequalities

Finite element method

Spectral methods

Methodology

Numerical analysis of PDEs

Scientific English

Labour legislation and ethics

Semester 4:

Research work sanctioned by a dissertation and a defense with the possibility of

Internship in an engineering company

III - Detailed program

Semester 1 – Core Unit (UEF1)

Master Title: PDEs and Applications Course Unit Title: UEF1 Subject Title: Distributions and Fourier Analysis Credits: 4 Coefficient: 3

Course Objectives:

To become familiar with the Schwartz space of infinitely differentiable functions and various concepts related to distributions. The course also covers differentiation in the sense of distributions and the convolution product.

Recommended Prerequisite Knowledge:

Functional spaces, integration theory.

Course Content:

I. Basic functions and distributions
II. Differentiation of distributions: Definitions, properties, examples
III. Convolution of distributions: Definitions, properties, examples
IV. Space of tempered distributions
V. Fourier transform of distributions S'\mathcal{S}'S'
VI. Laplace transform

Assessment Method:

 $\label{eq:constraint} $$ (2\times Final Exam Grade+Personal Work Grade)\times 13 \left[t(2 \times Final Exam Grade) + text{Personal Work Grade} \right] \\ frac{1}{3}(2\times Final Exam Grade+Personal Work Grade)\times 31 \\$

References:

- Claude Gasquet & Patrick Witomski, *Analyse de Fourier et Applications*, Masson, Paris, 2000
- Cohen and R.D. Ryan, Fourier Analysis and Wavelets, Chapman and Hall, 2000
- B. Torresani, *Analyse continue par ondelettes*, Inter Éditions, CNRS Éditions, Paris, 1995
- W. Appel, Mathématiques pour la physique, H&K, 2002

Master Title: PDEs and Applications Course Unit Title: UEF1 Subject Title: Distributions and Fourier Analysis Credits: 4 Coefficient: 3

Course Objectives:

To become familiar with the Schwartz space of infinitely differentiable functions and various concepts related to distributions. The course also covers differentiation in the sense of distributions and the convolution product.

Recommended Prerequisite Knowledge:

Functional spaces, integration theory.

Course Content:

I. Basic functions and distributions
II. Differentiation of distributions: Definitions, properties, examples
III. Convolution of distributions: Definitions, properties, examples
IV. Space of tempered distributions
V. Fourier transform of distributions S'\mathcal{S}'S'
VI. Laplace transform

Assessment Method:

 $\begin{array}{l} (2 \times Final Exam Grade+Personal Work Grade) \times 13 \left(2 \times Final Exam Grade \right) \\ + \left(2 \times Final Exam Grade \right) \\ frac{1}{3}(2 \times Final Exam Grade+Personal Work Grade) \\ \times 31 \end{array}$

References:

- Claude Gasquet & Patrick Witomski, *Analyse de Fourier et Applications*, Masson, Paris, 2000
- Cohen and R.D. Ryan, Fourier Analysis and Wavelets, Chapman and Hall, 2000
- B. Torresani, *Analyse continue par ondelettes*, Inter Éditions, CNRS Éditions, Paris, 1995
- W. Appel, Mathématiques pour la physique, H&K, 2002

Semester 1 – Core Unit (UEF2)

Subject Title: Mathematical Modeling Credits: 4 Coefficient: 2

Course Objectives:

- 1. Introduce students to modeling of real and complex phenomena (in biology, ecology, epidemiology, economics) using differential equations.
- 2. Develop skills enabling students to apply this approach in their final theses and provide a basis for future research.

Recommended Prerequisite Knowledge:

Differential equations and basic concepts of dynamical systems.

Course Content:

- General introduction to mathematical modeling
- Mathematical models in biology and ecology (population dynamics, interaction between two populations, community models)
- Epidemiological models (SI, SIR, SEIR, etc.)
- Economic models (sequential adjustment, cobweb models, etc.)

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

- V. Volterra, *Leçons sur la théorie mathématique de la lutte pour la vie*, Gauthier-Villars, 1931 (reprinted 1990)
- J. Istas, Introduction aux modélisations mathématiques pour les sciences du vivant, Mathématiques & Applications 34, 2000
- O. Diekmann & J.A.P. Heesterbeek, *Mathematical Epidemiology of Infectious Diseases*

Semester 1 – Methodological Unit (UEM1)

Subject Title: Matrix Analysis **Credits:** 5 **Coefficient:** 2

Course Objectives:

The goal is to deepen knowledge of numerical matrix analysis acquired in the undergraduate curriculum and provide students with the tools necessary for the numerical solution of PDEs, with emphasis on stability and convergence.

Recommended Prerequisite Knowledge:

Numerical Analysis II

Course Content:

- 1. Review and additional concepts in linear algebra and matrices
- 2. Solution of large linear systems: iterative methods (Jacobi, Gauss-Seidel, Conjugate Gradient)
- 3. Computation of eigenvalues and eigenvectors
- 4. Numerical solution of nonlinear systems
- 5. Programming: matrix data structures, sparse matrix storage, MATLAB programming

Assessment Method:

 $\begin{array}{l} (2 \times Final \ Exam \ Grade + Personal \ Work \ Grade) \times 13 \setminus left(2 \setminus times \setminus text{Final \ Exam \ Grade} + \\ + \left\{ Personal \ Work \ Grade \right\} \setminus times \\ \int frac{1}{3}(2 \times Final \ Exam \ Grade + Personal \ Work \ Grade) \times 31 \end{array}$

References:

- P. Lascaux, R. Théodor, *Analyse numérique matricielle appliquée à l'art de l'ingénieur*, Dunod, 2000
- J.P. Nougier, Méthodes de calcul numérique, Vol. 1, Hermes Science, 2001
- A. Ralston, P. Rabinowitz, *A First Course in Numerical Analysis*, International Student Edition, 1978

Semester 1 – Methodological Unit (UEM2)

Subject Title: MATLAB – PDE Toolbox **Credits:** 5 **Coefficient:** 2

Course Objectives: To improve students' proficiency with MATLAB and its PDE Toolbox.

Recommended Prerequisite Knowledge:

Numerical analysis, matrix analysis, MATLAB

Course Content:

- Study of MATLAB: vectors and matrices, complex numbers, polynomials, special functions, programming, graphs
- Applications: numerical analysis, probability and statistics
- PDE Toolbox: meshing, finite difference method, etc.

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

• M. Mokhtari, A. Mesbah, *Apprendre et maîtriser MATLAB*, Springer-Verlag, 1997

Semester 1 – Discovery Unit (UED1)

Subject Title: Scientific English Credits: 2 Coefficient: 1

Course Objectives:

To develop students' oral skills in scientific English, especially note-taking and active participation in scientific presentations in the field of mathematics.

Course Content (Selected Topics):

- Arithmetic operations, math symbols
- Word problems and math jargon
- Common and decimal fractions, percentages, ratios, averages
- Geometry and measurements
- Lecture comprehension and scientific terminology
- Grammar for mathematical use: tenses, modals, passive/active, relative clauses, conditionals

Assessment Method:

Final Exam

Semester 2 – Core Unit (UEF3)

Subject Title: Sobolev Spaces Credits: 5 Coefficient: 3

Course Objectives:

To study Sobolev spaces and their various properties. These are the natural functional spaces used for solving variational formulations of partial differential equations (PDEs).

Recommended Prerequisite Knowledge:

Integration theory and distribution theory.

Course Content:

- $H1(\Omega),Hm(\Omega),H-m(\Omega)H^{1}(Omega), H^m(Omega), H^{-m}(Omega), H^{-m}(Omega), H^{-m}(\Omega)$
- Poincaré inequality
- Density theorems
- Trace theorems and Green's formulas
- Sobolev gluing theorem
- Sobolev spaces via Fourier transform
- Fractional Sobolev spaces
- Sobolev embedding theorems and Rellich's theorem
- Weighted Sobolev spaces Wpm(Ω)W_p^m(\Omega)Wpm(Ω)

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

- R. Adams, Sobolev Spaces, Eilenberg-Bass, 1970
- R. Dautray & J.L. Lions, *Mathematical Analysis and Numerical Calculation*, Masson, 1988
- G. Allaire, *Numerical Analysis and Optimization*, Éditions de l'École Polytechnique, 2009
- H. Brézis, *Functional Analysis*, Masson, 1993
- P.A. Raviart & J.M. Thomas, *Introduction to the Numerical Analysis of PDEs*, Masson, 1983

Semester 2 – Core Unit (UEF3)

Subject Title: Evolutionary PDEs **Credits:** 5 **Coefficient:** 2

Course Objectives:

To introduce students to semigroup theory and its applications to time-dependent PDEs, especially heat, wave, and Schrödinger equations. Special attention is given to Stokes and Navier-Stokes systems.

Recommended Prerequisite Knowledge:

Functional analysis (S1), applied functional analysis (S2).

Course Content:

- Evolutionary PDEs: heat, wave, Schrödinger equations
- Dissipative and m-dissipative operators in Hilbert spaces
- Strongly continuous semigroups, Hille-Yosida and Lumer-Phillips theorems
- Cauchy problems and semigroup solutions

Assessment Method:

Same formula as above.

References:

- E. Davies, *One Parameter Semigroups*, Academic Press, 1980
- Girault & P.A. Raviart, *Finite Element Approximation of Navier-Stokes*, LN Math 749, Springer, 1979
- A. Pazy, Semigroups of Linear Operators and Applications to PDEs, Springer, 1983

Semester 2 – Core Unit (UEF4)

Subject Title: Numerical Analysis of ODEs **Credits:** 5 **Coefficient:** 2

Course Objectives:

To describe various one-step and multi-step numerical methods to solve initial value problems for ordinary differential equations (ODEs). Lab sessions focus on implementing the studied algorithms.

Recommended Prerequisite Knowledge:

ODEs and numerical integration methods.

Course Content:

- Review of ODEs and Cauchy problem
- Cauchy-Lipschitz theorem
- Solution regularity
- One-step methods: Euler, midpoint, Taylor of order ppp
- Concepts of consistency, stability, convergence
- Runge-Kutta methods
- Multi-step methods: Adams-Bashforth, Adams-Moulton, predictor-corrector methods

Assessment Method:

Same as above.

References:

- M. Crouzeix & A.L. Mignot, *Numerical Analysis of Differential Equations*, Masson, 1984 & 1986
- J.-P. Demailly, *Numerical Analysis and Differential Equations*, P.U. Grenoble, 1996

Semester 2 – Core Unit (UEF2)

Subject Title: Introduction to Unbounded Operators Credits: 3 Coefficient: 2

Course Objectives:

To familiarize students with theoretical concepts necessary for spectral theory in PDEs, focusing on eigenvalues and eigenfunctions of differential operators.

Recommended Prerequisite Knowledge:

Topology of metric spaces, Hilbert space analysis.

Course Content:

- Compact operators, kernel operators, Hilbert-Schmidt operators (examples from integral operators)
- Unbounded operators: closed operators, adjoints, self-adjointness, resolvent and spectrum
- Spectral theory of compact and resolvent-compact self-adjoint operators

Same as above.

References:

- H. Brézis, Functional Analysis, Masson, 1993
- A.-S. Bonnet-Bendhia & P. Joly, *Spectral Theory of Self-Adjoint Operators*, course notes
- El Kacimi, Elements of Integration and Functional Analysis, Ellipses, 1999
- S. Maingot & D. Manceau, Spectral Theory, course notes

Semester 2 – Methodological Unit (UEM2)

Subject Title: Introduction to Variational Approximation Methods **Credits:** 4 **Coefficient:** 2

Course Objectives:

This module aims to provide a variational formulation for boundary value problems, proving the existence and uniqueness of weak solutions, while offering a framework well suited to numerical approximation.

Recommended Prerequisite Knowledge:

Operators in Hilbert spaces, Sobolev spaces, and Green's formulas.

Course Content:

- Lax-Milgram Theorem, generalized Poincaré inequality
- Variational formulation of elliptic boundary value problems:
 - Laplace equation with Dirichlet, Neumann, and mixed boundary conditions
- General internal variational approximation:
 - Transition from continuous to discrete variational problems
 - Céa's Lemma, convergence of the method, Galerkin method

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

• G. Allaire, *Numerical Analysis and Optimization*, Éditions de l'École Polytechnique, 2009

• P.A. Raviart & J.M. Thomas, *Introduction to Numerical Analysis of PDEs*, Masson, 1983

Semester 2 – Methodological Unit (UEM2)

Subject Title: Computer Tools Credits: 4 Coefficient: 1

Course Objectives:

Maple is a symbolic computation software encompassing all areas of modern mathematics, including some of its most advanced aspects. The course's goal is to teach students how to use and master this tool.

Recommended Prerequisite Knowledge:

Basic knowledge of programming (Pascal, Fortran)

Course Content: Part I – Introduction to Maple:

- 1. First steps: basic commands, numeric & symbolic calculations, basic functions, exercises
- 2. Arithmetic: complex numbers, polynomial operations, equation solving
- 3. Graphics: 2D/3D plots, animations, exercises
- 4. Maple data types: sequences, lists, sets, arrays, tables
- 5. Linear algebra: vector/matrix operations, solving linear systems

Part II – Introduction to LaTeX:

- 1. Basics of LaTeX
- 2. Installation and usage
- 3. Mathematical modes in LaTeX
- 4. Writing a document
- 5. Creating Beamer presentations

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

• Lionel Porcheron, *Maple*, Dunod, 2006

- Jean-Michel Ferrard, Maths et Maple, Dunod, 1998
- Ibrahim Alane & Patrick Le Quéré, *Maple V.4*, Eyrolles, 1997

Semester 2 – Discovery Unit (UED2)

Subject Title: Analytical Mechanics **Credits:** 2 **Coefficient:** 2

Course Objectives:

To provide an overview of various mathematical formulations of mechanical systems (Newtonian, Lagrangian, and Hamiltonian formulations), showing the deep link between mechanics and mathematics.

Recommended Prerequisite Knowledge:

Point particle mechanics, vector analysis

Course Content: Chapter 1 – Newtonian Mechanics:

- Particle mechanics, multi-particle systems, rigid body mechanics Chapter 2 – Lagrangian Formalism:
- Variational principle, Lagrange's equations, holonomic and non-holonomic constraints, Lagrange multipliers
 - **Chapter 3 Hamiltonian Formalism:**
- Legendre transform, Hamilton's equations, canonical variables, Poisson brackets, generalized momenta, Hamilton-Jacobi theory, phase space, action-angle variables, integrable systems

Assessment Method:

 $\begin{array}{l} (2 \times Final Exam Grade+Personal Work Grade) \times 13 \\ + \left(2 \times Final Exam Grade\right) \\ + \left(2 \times Final Exam Grade\right) \\ frac{1}{3}(2 \times Final Exam Grade+Personal Work Grade) \\ \times 31 \end{array}$

References:

- Landau & Lifshitz, Mechanics, Mir & Ellipses
- H. Goldstein et al., Classical Mechanics, 3rd ed., Addison-Wesley, 2001
- L. Marleau & P. Amiot, *Classical Mechanics*, Vol. 1 & 2

Semester 2 – Discovery Unit (UED2)

Subject Title: Scientific English Credits: 1 Coefficient: 1

Course Objectives:

To develop students' proficiency in scientific and business English through simulations of scientific conferences, poster creation, CV and cover letter writing, mock interviews, role-playing (start-up creation), blogging, and intercultural projects.

Recommended Prerequisite Knowledge:

Basic English

Course Content: Not explicitly detailed.

Assessment Method: Final Exam

References: Audio support

Semester 3 – Core Unit (UEF5)

Subject Title: Variational Analysis of PDEs **Credits:** 5 **Coefficient:** 3

Course Objectives:

This module focuses on elliptic boundary value problems commonly encountered in mechanics and physics. The main objective is to formulate them variationally, demonstrate the existence and uniqueness of weak solutions, and study their regularity.

Recommended Prerequisite Knowledge:

Sobolev spaces, Introduction to variational approximation methods

Course Content:

I. Variational formulation of elliptic problems:

- Existence and uniqueness via Lax-Milgram Theorem (review)
- Laplacian with non-homogeneous Dirichlet boundary conditions
- Laplacian with non-homogeneous Neumann boundary conditions
- Dirichlet problem for the biharmonic operator
- Transmission problems

II. Variational approach to linear elasticity system

III. Variational approach to Stokes system

IV. Regularity of solutions to variational problems

Assessment Method:

 $\begin{array}{l} (2 \times Final \ Exam \ Grade + Personal \ Work \ Grade) \times 13 \setminus left(2 \setminus times \ text{Final \ Exam \ Grade} + \\ + \ text{Personal \ Work \ Grade} \setminus times \\ \int frac{1}{3}(2 \times Final \ Exam \ Grade + Personal \ Work \ Grade) \times 31 \end{array}$

References:

- G. Allaire, *Numerical Analysis and Optimization*, Éditions de l'École Polytechnique, 2005
- R. Dautray & J.L. Lions, *Mathematical Analysis and Numerical Calculation*, Masson, 1988
- J. Necas, Direct Methods in the Theory of Elliptic Equations, Masson, 1967
- P.A. Raviart & J.M. Thomas, *Introduction to Numerical Analysis of PDEs*, Masson, 1983

Subject Title: Variational Inequalities **Credits:** 4 **Coefficient:** 2

Course Objectives:

This course complements variational analysis of PDEs and allows students to master variational inequalities used in modeling PDEs with multivalued boundary conditions.

Recommended Prerequisite Knowledge:

Functional analysis, operator theory, evolution equations

Course Content:

- Variational inequalities
- Basic existence and uniqueness results
- Convergence results
- Quasi-variational inequalities
- Evolutionary variational inequalities
- Existence, uniqueness, and regularization
- Applications:
 - Friction problems
 - Economic planning models

Assessment Method:

(2×Final Exam Grade+Personal Work Grade)×13\left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade}\right) \times \frac{1}{3}(2×Final Exam Grade+Personal Work Grade)×31

References:

- H. Brézis, Variational Inequalities Associated with Evolution Operators, NATO Institute, Venice, 1968
- H. Brézis, Abstract Evolution Inequalities, C.R. Acad. Sci., 264, 1967
- J.-B. Hiriart-Urruty & C. Lemaréchal, *Convex Analysis and Minimization Algorithms I & II*, Springer, 1993

Semester 3 – Core Unit (UEF6)

Subject Title: Finite Element Method **Credits:** 5 **Coefficient:** 3

Course Objectives:

Study of the finite element method in 1D and 2D: method presentation, convergence, and error estimation.

Recommended Prerequisite Knowledge:

Functional analysis, Sobolev spaces, matrix analysis

Course Content:

- Finite element method in one spatial variable
- Finite element method in two spatial variables
- Interpolation operators, approximation error, and convergence

Assessment Method:

 $\label{eq:constraint} \begin{array}{l} (2\times Final Exam Grade+Personal Work Grade)\times 13 \left(2 \times \text{Final Exam Grade} + \text{Personal Work Grade} \times \frac{1}{3}(2\times Final Exam Grade+Personal Work Grade)\times 31 \end{array}$

References:

- G. Allaire, *Numerical Analysis and Optimization*, Éditions de l'École Polytechnique, 2009
- Ciarlet, The Finite Element Method for Elliptic Problems, North Holland, 1978
- P.A. Raviart & J.M. Thomas, *Introduction to Numerical Analysis of PDEs*, Masson, 1983

Semester 3 – Core Unit (UEF6)

Subject Title: Spectral Methods Credits: 4 Coefficient: 2

Course Objectives:

This course presents spectral methods as tools for approximating PDEs. Focus is placed on the Dirichlet problem for the Laplacian on the square $[-1,1]2[-1,1]^2[-1,1]2$ or the cube $[-1,1]3[-1,1]^3[-1,1]3$.

Recommended Prerequisite Knowledge:

Sobolev spaces, orthogonal projection theorem, polynomial interpolation, Lax-Milgram theorem, Dirichlet boundary problem for the Laplacian

Course Content:

- Legendre polynomials and quadrature formulas
 - Gauss and Gauss-Lobatto quadratures
- Polynomial approximation and interpolation errors
 - o 1D and higher dimensions
- Spectral discretization of the Dirichlet problem for the Laplacian

Final Exam (coefficient 2) + Personal Work (coefficient 1)

References:

- C. Bernardi & Y. Maday, Spectral Approximations of Elliptic Boundary Value Problems, Springer, 1992
- C. Bernardi & Y. Maday, Spectral Methods, North-Holland, 1997
- C. Bernardi, Y. Maday, F. Rapetti, Variational Discretizations of Elliptic Boundary Problems, Springer, 2004

emester 3 – Methodological Unit (UEM3)

Subject Title: Teaching Methodology Credits: 3 Coefficient: 1

Course Objectives:

To train students in teaching methods adapted to promote learning and achieve pedagogical goals.

Recommended Prerequisite Knowledge:

(Not specified)

Course Content:

- General concepts and definitions: methodology, teaching, education, school, teacher; distinctions between these terms
- Teaching as an art
- Teaching as a profession
- Fundamentals and principles of teaching
- Teaching and training competencies
- Evolution and stages of teaching competencies
- Planning and preparation for teaching
- Teaching practices
- Teaching procedures and methods

- Efficient teaching and effective teachers
- Teaching methodology in the context of modern technologies

Final Exam

References:

(Not specified)

Semester 3 – Methodological Unit (UEM3)

Subject Title: Numerical Analysis of PDEs **Credits:** 6 **Coefficient:** 2

Course Objectives:

This course introduces modeling through the physical origin of various elliptic, parabolic, and hyperbolic PDEs, and presents methods for their numerical resolution (consistency, stability, convergence).

Recommended Prerequisite Knowledge:

Mathematical physics equations, finite difference methods

Course Content: I. Elliptic Problems:

- Modeling
- Discretization strategies: finite difference and finite volume methods (1D & 2D)
- Numerical analysis: approximate solution properties, convergence
- Application to Sturm-Liouville problems

II. Scalar Hyperbolic Problems:

- Modeling
- Approximation methods: centered, upwind, Lax-Friedrichs schemes
- Stability analysis of different schemes

III. Parabolic Problems:

- Modeling
- Finite difference schemes for the heat equation (explicit/implicit Euler, Crank-Nicolson for time)
- Numerical analysis

 $\label{eq:action} \begin{array}{l} (2\times Final \ Exam \ Grade+Personal \ Work \ Grade)\times 13 \ ft(2 \ times \ text{Final \ Exam \ Grade} \\ + \ text{Personal \ Work \ Grade} \ times \\ \ frac{1}{3}(2\times Final \ Exam \ Grade+Personal \ Work \ Grade)\times 31 \end{array}$

References:

- F. Hubert, J. Hubbard, *Scientific Computing: From Theory to Practice*, Vuibert, 2006
- L. Sainsaulieu, Scientific Computing, Masson, 1996
- M. Schatzmann, *Numerical Analysis*, Interéditions, 1991
- D. Serre, Systems of Conservation Laws, Vol. I & II, Cassini, 1996

Semester 3 – Transversal Unit (UET3)

Subject Title: Scientific English Credits: 2 Coefficient: 1

Course Objectives:

To develop students' oral communication skills in scientific English, especially notetaking and participating in scientific presentations in mathematical sciences.

Course Content:

- Arithmetic operations, math symbols
- Word problems and mathematical terminology
- Fractions and decimals, percentages
- Ratios, averages, measurements (length, mass, volume)
- Geometry terms (angle, area, right angle, etc.)
- Understanding fast speech in lectures
- Common homophones: angle/ankle, pair/pear, eight/ate
- Grammar:
 - Conditionals (zero, first)
 - Present simple/continuous, past simple, present perfect
 - Comparatives, ING forms (thinking, reasoning), modal verbs
 - Infinitives, past participles, passive/active forms
 - Relative clauses, reported speech
 - Regular/irregular nouns, verbs, adjectives

Assessment Method:

Final Exam

Semester 3 – Discovery Unit (UED3)

Subject Title: Labor Law and Professional Ethics **Credits:** 1 **Coefficient:** 1

Course Objectives:

To inform and raise awareness among students about corruption risks and encourage them to participate in anti-corruption efforts.

Recommended Prerequisite Knowledge:

(Not specified)

Course Content:

- Concept of corruption
- Types and manifestations of administrative and financial corruption
- Theoretical and general causes of corruption
- Effects of corruption
- Local and international anti-corruption efforts
- Methods and tools to combat corruption
- International models and experiences

Assessment Method:

Final Exam

References:

(Not specified)

Semester 4 – Thesis Project

Subject Title: Master's Thesis Credits: 30 Coefficient: –

Objectives:

The fourth semester is entirely dedicated to the student's **research or applied project**, culminating in the writing and **defense of a Master's thesis**. The aim is to allow the student to:

- Deepen theoretical and/or practical knowledge in a chosen area
- Apply mathematical modeling, numerical methods, or control theories to real problems
- Develop autonomy in research, documentation, critical analysis, and scientific communication
- Prepare for either **PhD studies** or direct integration into the **R&D departments** of companies or public institutions

Assessment Method:

- Written Thesis
- Oral Defense before a jury

Supervision and Evaluation:

Each student is supervised by an academic advisor (and possibly an industrial co-advisor) and evaluated by a jury composed of at least three members. The evaluation takes into account:

- Scientific quality and originality
- Methodological rigor
- Relevance of results
- Writing quality and clarity
- Oral presentation and defense